Selected Financial Formulae

Purpose	Formula
Basic Time Value Formulae	
Future Value of a Single Sum	$FV = PV(1+i)^N$
Present Value of a Single Sum	$PV = \frac{FV}{(1+i)^N}$
Solve for N for a Single Sum	$N = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)}$
Solve for i for a Single Sum	$i = \sqrt[N]{\frac{FV}{PV}} - 1$
Present Value of an Ordinary Annuity	$PV_A = Pmt\left[\frac{1 - 1/(1+i)^N}{i}\right]$
Future Value of an Ordinary Annuity	$FV_A = Pmt\left[\frac{(1+i)^N - 1}{i}\right]$
Present Value of an Annuity Due	$PV_{Ad} = Pmt\left[\frac{1 - 1/(1+i)^{(N-1)}}{i}\right] + Pmt$
Future Value of an Annuity Due	$FV_{Ad} = Pmt\left[\frac{(1+i)^N - 1}{i}\right](1+i)$
Basic Security Valuation Formulae	
Dividend Discount Model (AKA, the Gordon Model)	$V_{CS} = \frac{D_0(1+g)}{k_{CS} - g} = \frac{D_1}{k_{CS} - g}$
Two-stage Dividend Discount Model Notes: This equation is too long for one line. g_1 = Growth rate during high growth phase. g_2 = Growth in constant growth phase after n. n = Length of high growth phase. Assume $g_1 <> k_{CS}$ and $g_2 < k_{CS}$	$V_{CS} = \frac{D_0(1+g_1)}{k_{CS} - g_1}\left[1 - \left(\frac{1+g_1}{1+k_{CS}}\right)^n\right] + \frac{\frac{D_0(1+g_1)^n(1+g_2)}{k_{CS} - g_2}}{(1+k_{CS})^n}$

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Three-stage Dividend Discount Model Notes: n_1 = Length of high growth phase. n_2 = Periods until constant growth phase. $n_2 = n_1$ + length of transistion phase.	$V_{CS} = \frac{D_0}{k_{CS} - g_2}\left[(1 + g_2) + \frac{n_1 + n_2}{2}(g_1 - g_2)\right]$
Earnings Model	$V_{CS} = \frac{EPS_1}{k_{CS}} + \frac{RE_1\left(\frac{ROE}{k_{CS}} - 1\right)}{k_{CS} - g}$
Constant Growth FCF Valuation Model V_{Ops} = Value of Total Operations V_{Debt}, V_{Pref} = Value of debt and preferred stock $V_{Non\text{-}Ops\ Assets}$ = Value of non-operating assets	$V_{Ops} = \frac{FCF_1}{k_{CS} - g}$ $V_{CS} = V_{Ops} - V_{Debt} - V_{Pref} + V_{Non-OpAssets}$
Sustainable growth rate Note: b = retention ratio = 1 - payout ratio r = return on equity	$g = br$
Value of a Share of Preferred Stock	$V_P = \frac{D}{k_P}$
Value of a Bond on a Payment Date	$V_B = Pmt\left[\frac{1 - 1/(1 + k_d)^N}{k_d}\right] + \frac{FV}{(1 + k_d)^N}$
Quoted Price of a Bond on a Non-Payment Date $V_{B,0}$ = Value of bond at last payment date α = The fraction of the current period that has elaspsed	$V_{B,\alpha} = V_{B,0}(1 + k_d)^{\alpha} - \alpha(Pmt)$
Basic Statistical Formulae	
Arithmetic Mean (Average)	$\bar{X} = \frac{1}{N}\sum_{t=1}^{N} X_t$
Geometric Mean (used for averaging returns, growth rates, etc.)	$\bar{G} = \sqrt[N]{\prod_{t=1}^{N}(1 + R_t)} - 1$

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Expected Value (Weighted Average)	$E(X) = \sum_{t=1}^{N} \rho_t X_t$
Variance	$\sigma_X^2 = \sum_{t=1}^{N} \rho_t (X_t - \bar{X})^2$
Standard Deviation	$\sigma_X = \sqrt{\sigma_X^2}$
Covariance	$\sigma_{X,Y} = \sum_{t=1}^{N} [\rho_t (X_t - \bar{X})(Y_t - \bar{Y})]$
Correlation Coefficient	$r_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$
Beta (Note: *M* is the market portfolio, and *i* is the security or portfolio)	$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2} = \frac{r_{i,M} \sigma_i \sigma_M}{\sigma_M^2}$
Portfolio Formulae	
Expected Return of a Portfolio	$E(R_P) = \sum_{i=1}^{N} w_i R_i$
Variance of a 2-security Portfolio	Using the covariance: $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{1,2}$ or, using the correlation coefficient: $\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 r_{1,2} \sigma_1 \sigma_2$
Variance of an N-security portfolio Using the Covariance	$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{i,j}$
Standard Deviation of a Portfolio	$\sigma_P = \sqrt{\sigma_P^2}$

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Portfolio Beta	$\beta_P = \sum_{i=1}^{N} w_i \beta_i$
95% Value at Risk (Variance/Covariance Model) Note: V_p is portfolio value	$\text{VaR} = 1.645 \times V_p \times \sigma_p$
Capital Market Theory Models	
Capital Market Line (CML)	$E(R_P) = R_f + \sigma_P \frac{(E(R_M) - R_f)}{\sigma_M}$
Capital Asset Pricing Model (CAPM) Note: This is also the equation for the Security Market Line (SML)	$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$
Treynor's Risk-adjusted Performance Measure	$T_i = \frac{R_i - R_f}{\beta_i}$
Sharpe's Risk-adjusted Performance Measure	$S_i = \frac{R_i - R_f}{\sigma_i}$
The Information Ratio	$\text{IR}_P = \frac{R_P - R_B}{\sigma_{R_P - R_B}}$
M^2 (Modigliani & Modigliani) Performance Measure	$M^2 = \left(\frac{\sigma_m}{\sigma_i}\right)(R_i - R_f) + R_f$
Fama's Risk Decomposition Notes: R_i = Portfolio Return R_M = Market Return R_f = Risk-free Rate β_i = Portfolio Beta β_T = Target Beta	$\text{Risk Premium} = R_i - R_f$ $\text{Risk} = \beta_i(R_M - R_f)$ $\text{Selectivity} = \text{Risk Premium} - \text{Risk}$ $\text{Managers Risk} = (\beta_i - \beta_T)(R_M - R_f)$ $\text{Investors Risk} = \beta_T(R_M - R_f)$ $\text{Diversification} = \left(\frac{\sigma_i}{\sigma_M} - \beta_i\right)(R_M - R_f)$ $\text{Net Selectivity} = \text{Selectivity} - \text{Diversification}$

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Brinson, Hood, and Beebower Additive Attribution Model Notes: A_t = Overall Allocation Effect S_t = Overall Selection Effect I_t = Overall Interaction Effect $w_{i,t}$ = Weight of Sector i in portfolio t bars over variables represent benchmark weights or returns.	$A_t = \sum_{i=1}^{N} (w_{i,t} - \bar{w}_{i,t})(\bar{R}_{i,t} - \bar{R}_t)$ $S_t = \sum_{i=1}^{N} \bar{w}_{i,t}(R_{i,t} - \bar{R}_{i,t})$ $I_t = \sum_{i=1}^{N} (w_{i,t} - \bar{w}_{i,t})(R_{i,t} - \bar{R}_{i,t})$
Options and Futures Valuation Models	
Black-Scholes European Call Option Valuation Model	$C = SN(d_1) - Xe^{-rt}N(d_2)$ where: $d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + 0.5\sigma^2)t}{\sigma\sqrt{t}}$ $d_2 = d_1 - \sigma\sqrt{t}$
Black-Scholes European Put Option Valuation Model (see above for d_1 and d_2)	$P = Xe^{-rt}N(-d_2) - SN(-d_1)$
Put-Call Parity for European Options with No Cash Flows	$C = P + S - Xe^{-rt}$ or, $P = C + Xe^{-rt} - S$
Single-period Binomial Option Pricing Model for Call Options (r is the risk-free rate, u is the up factor, and d is the down factor)	$C = \frac{pC_u + (1-p)C_d}{(1+r)}$ where, $p = \frac{r-d}{u-d}$
Single-period Binomial Option Pricing Model for Put Options	$P = \frac{pP_u + (1-p)P_d}{(1+r)}$ where, $p = \frac{r-d}{u-d}$

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Cost of Carry Model for Pricing Futures Contracts (CC is the carrying costs as a % of the spot price)	${}_{T}F_0 = S_0 e^{CC(t)}$
Bond Analysis Formulae	
Macaulay's Duration on a Payment Date (for immunization). Note: C_t is the cash flow in period t, i is the yield to maturity	$D = \dfrac{\sum_{t=1}^{N} \dfrac{C_t(t)}{(1+i)^t}}{\text{Bond Price}}$
Modified Duration (for price volatility) on a Payment Date	$D_{Mod} = \dfrac{D}{(1+i)}$
Convexity on a Payment Date	$C = \dfrac{\dfrac{1}{(1+i)^2}\left[\sum_{t=1}^{N} (t^2+t)\dfrac{Cf_t}{(1+i)^t}\right]}{\text{Bond Price}}$
The n-period forward rate given two spot rates (note that i > j, and n = i - j)	${}_{t+j}R_n = \sqrt[n]{\dfrac{(1+R_i)^i}{(1+R_j)^j}}$
Bank Discount Yield for discount securities (FV = face value, PP = purchase price, m = periods per year)	$BDY = \dfrac{FV-PP}{FV} \times \dfrac{360}{m}$
Bond Equivalent Yield for discount securities (see definitions for BDY)	$BEY = \dfrac{FV-PP}{PP} \times \dfrac{365}{m} = BDY \times \dfrac{FV}{PP} \times \dfrac{365}{360}$
Note: The Bond Analysis Formulae do not apply only to bonds. They may be used with any stream of cash flows.	
Capital Budgeting Decision Formulae	
Net Present Value (*NPV*)	$NPV = \sum_{t=1}^{N} \dfrac{Cf_t}{(1+i)^t} - IO$
Profitability Index (*PI*)	$PI = \dfrac{\sum_{t=1}^{N} \dfrac{Cf_t}{(1+i)^t}}{IO} = \dfrac{NPV+IO}{IO} = \dfrac{NPV}{IO} + 1$

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Internal Rate of Return (*IRR*). Note: This is a trial and error procedure to find the *i* that makes the equality hold (i.e., what discount rate makes the *NPV* = 0).	$0 = \sum_{t=1}^{N} \frac{Cf_t}{(1+i)^t} - IO$
Modified Internal Rate of Return (*MIRR*).	$MIRR = \sqrt[N]{\frac{\sum_{t=1}^{N} Cf_t(1+i)^{(N-t)}}{IO}} - 1$
Stock Market Index Construction Formulae	
Price-weighted Average (e.g., DJIA) Note: The divisor (Div) at period 0 is equal to the number of stocks in the average. It will be adjusted for stock splits or any other corporate action that results in a non-economic change in the stock price.	$PWA_t = \frac{\sum_{j=1}^{N} P_j}{Div_t}$
Capitalization-weighted Index (e.g., S&P 500) Note: The divisor (Div) at period 0 is the divisor that makes the initial level of the index equal to the desired starting point. It will be adjusted for any corporate action that results in a change in market capitalization.	$CWI_t = \frac{\sum_{j=1}^{N} P_j Q_j}{Div_t}$
Equally-weighted Arithmetic Index (e.g., VLA) Note: At period 0 the index is set to some starting value (e.g., 100). To calculate the index for any day, multiply the average % change by the previous index level.	$EWAI_t = EWAI_{t-1} \times \sum_{j=1}^{N} \left(\frac{P_{j,t}}{P_{j,t-1}}\right) / N$
Equally-weighted Geometric Index (e.g., VLG) Note: See note above	$EWGI_t = EWGI_{t-1} \times \sqrt[N]{\prod_{j=1}^{N} \frac{P_{j,t}}{P_{j,t-1}}}$
Corporate Financial Formulae	
Net Operating Profit After Taxes (NOPAT)	$\text{NOPAT} = \text{EBIT}(1-t)$
Net Operating Working Capital (NOWC)	NOWC = Op. C.A. – Op. C.L.

Selected Financial Formulae

Purpose	Formula
Operating Capital (Op. Cap.)	$\text{Op. Cap.} = \text{NOWC} + \text{NFA}$
Free Cash Flow (FCF)	$\text{FCF} = \text{NOPAT} - \text{Net Investment in Op. Cap.}$
Economic Value Added (EVA)	$\text{EVA} = \text{NOPAT} - (\text{Op. Cap.} \times \text{Cost of Cap.})$
Miscelaneous Formulae	
Margin Call Trigger Price Note: IM% is the initial margin supplied, MM% is the maintenance margin requirement, P_0 is the initial value of the portfolio	$P_M = \frac{\text{IM\%} - 1}{\text{MM\%} - 1} \times P_0$
Percentage gain to recover (% GTR) from a loss (%L)	$\text{\%GTR} = \frac{1}{1 - \text{\%L}} - 1$